THE BATSHEVA DE ROTHSCHILD SEMINAR ON QUASICRYSTALS, DELONE SETS AND GENERALIZATIONS OF LATTICES

Schedule

	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday
7:00 - 9:00		Breakfast	Breakfast	Breakfast	Breakfast	Breakfast
Morning Session		Sadun	Björklund	Marklof	Lev	Sadun
		Coffee break	Coffee break	Coffee break	Coffee break	Coffee break
		Marklof	Lev	Björklund	Sadun	Nevo
		Björklund	Sadun	Lev	Strömberg-sson	Takeout Lunch + Departure shuttles
12:30 - 14:30	Arrival shuttles	Lunch + discussions	Lunch + discussions	Lunch + excursion	Lunch + discussions	
14:30 - 16:00	Registration					
Afternoon Session	Pogorzelski	Gringlaz	Adiceam		Solomyak	
	Coffee break	Coffee break	Coffee break		Coffee break	
	TBA	Smilansky	Strömberg-sson		Björklund	
18:30 - 20:00	Dinner	Dinner	Dinner	Conference Dinner	Dinner	
20:30 - 22:00	Evening discussions	Evening discussions	Problem session		Evening discussions	

Detailed schedule:

Sunday,	11.03.18:
14:30 - 16:00	Arrival shuttles, registration
16:00 - 17:10	Pogorzelsky (70 minutes)
17:10 - 17:40	Coffee break
17:40 - 18:50	TBA (70 minutes)
18:50 - 20:00	Dinner
20:30 - 22:00	Evening discussions
Monday,	12.03.18:
07:00 - 09:00	Breakfast
09:00 - 10:00	Sadun
10:00 - 10:20	Coffee break
10:20 - 11:20	Marklof
11:30 - 12:30	Björklund
12:30 - 16:00	Lunch and discussions
16:00 - 17:00	Gringlaz
17:00 - 17:30	Coffee break
17:30 - 18:30	Smilansky
18:30 - 20:00	Dinner
20:30 - 22:00	Evening discussions
Tuesday,	13.03.18:
07:00 - 09:00	Breakfast
09:00 - 10:00	Björklund
10:00 - 10:20	Coffee break
10:20 - 11:30	Lev (70 minutes)
11:40 - 12:40	Sadun
12:40 - 16:00	Lunch and discussions
16:00 - 17:00	Adiceam
17:00 - 17:30	Coffee break
17:30 - 18:30	Strömbergsson
18:30 - 20:00	Dinner
20:30 - 22:00	Evening discussions

Wednesday,	14.03.18:
07:00 - 09:00	Breakfast
09:00 - 10:00	Marklof
10:00 - 10:20	Coffee break
10:20 - 11:20	Björklund
11:25 - 12:35	Lev (70 minutes)
12:35 - 13:30	Lunch
13:30 - 18:00	Excursion
18:30 - 21:00	Conference dinner
Thursday,	15.03.18:
07:00 - 09:00	Breakfast
09:00 - 10:10	Lev (70 minutes)
10:10 - 10:30	Coffee break
10:30 - 11:30	Sadun
11:40 - 12:40	Strömbergsson
12:40 - 16:00	Lunch and discussions
16:00 - 17:00	Solomyak
17:00 - 17:30	Coffee break
17:30 - 18:30	Björklund
18:30 - 20:00	Dinner
20:30 - 22:00	Evening discussions
Friday,	16.03.18:
07:00 - 08:30	Breakfast
08:30 - 09:30	Sadun
09:30 - 10:00	Coffee break
10:00 - 11:00	Nevo
11:15	Takeout lunch and departure shuttles

Abstracts:

Mini-courses:

<u>Mini-course 1:</u> **The space of quasicrystals and applications to mathematical physics**, Jens Marklof and Andreas Strömberg-sson.
We will survey recent work where we associate to a given quasicrystal P of cut-and-project type a certain homogeneous space X, which can be viewed as the closure of the family of all point sets obtained by acting on P with an arbitrary linear (or affine linear) map of determinant 1. The space X carries an invariant probability measure and the corresponding *random* cut-and-project set plays an important role in answering questions about certain asymptotic properties in the original set P, e.g. fine-scale statistics of directions to points in P visible from the origin, and free path lengths in a Lorentz gas with scatterer configuration given by P.

Lecture 1: Here we give the construction of X; its existence and basic properties hinges on Ratner's theorem on unipotent flows in homogeneous dynamics. We also introduce two standard topologies on the family of discrete point sets in $\mathbb{R}^d$, and show how the space X can be viewed as the closure of the $SL(d, R)$-orbit of P (as mentioned above).

Lecture 2: In this lecture we explain the Siegel-Veech formula for the *intensity* of the point process (the random cut-and-project set) described by X.

Lecture 3: Applications. Here we further discuss the space X and explain in particular how results in homogeneous dynamics lead to facts about statistics of directions to points in P and on free path lengths in a Lorentz gas with scatterer configuration P.

Lecture 4: In this lecture we discuss generalizations to other point sets P, highlighting the question whether it is possible to characterize which point processes in $\mathbb{R}^d$ are invariant and ergodic under the action of the affine special linear group.

<u>Mini-course 2:</u> **The topology of spaces of discrete point sets (and tilings)**, Lorenzo Sadun.
This series of lectures will introduce the topological tools, especially cohomology, used to study spaces of Delone sets, also known as tiling spaces.

In the first lecture I will define tiling spaces and their dynamics and explain the advantages of studying these spaces versus studying individual tilings or point patterns. We will see what tiling spaces look like locally, and how they can be expressed as inverse limits of branched manifolds.

In the second lecture I will introduce tiling cohomology and explain how cohomology is actually computed in a number of examples. There are several different versions of tiling cohomology, most notably Cech cohomology and pattern-equivariant cohomology. We will see how these versions are all isomorphic on the most common types of tiling spaces, although they can differ when certain standard assumptions are relaxed.

In the third lecture we will see how the first cohomology group H^1, taken with real coefficients, parametrizes deformations of tilings and helps to classify tiling spaces up to homeomorphism. This gets to the heart of understanding which properties of tilings are essentially combinatorial, and which depend on the geometry of the tiles themselves.

In the fourth and final lecture we will look at the top-dimensional cohomology group H^n. This group is associated with invariant measures on tiling spaces, which in turn is related to frequencies of different patterns that appear in a tiling. From the cohomology we can also derive estimates on the rate at which empirical patch frequencies on finite regions converge to their infinite-volume limit.

Mini-course 3: **Approximate lattices**, Michael Björklund.
The four lectures in the course will cover some basic properties of approximate lattices. The rough (and time-optimistic) outline will be as follows.

Lecture 1: We survey the classical theory of approximate lattices in Euclidean spaces developed by Yves Meyer, and point towards non-commutative generalizations.

Lecture 2: We discuss examples of (aperiodic) approximate lattices in non-commutative groups. In particular, we show that (an extension of) Meyers classical cut-and-project sets are approximate lattices, whose hulls admit unique invariant probability measures.

Lecture 3: We sample a a few results from the paper Approximate lattices by B. and Tobias Hartnick; in particular we show that non-unimodular groups admit no approximate lattices and that approximate lattices in nilpotent groups are uniform.

Lecture 4: We discuss the spectral theory of approximate lattices; this leads to the notion of diffraction - if time permits, we shall cover some non-commutative aspects of this theory, which have been developed jointly with Tobias Hartnick and Felix Pogorzelski.

Mini-course 4: **Fourier quasicrystals**, Nir Lev.
By a "Fourier quasicrystal" we mean a discrete distribution of masses that has a pure point spectrum. This notion was inspired by the experimental discovery in the 80's of non-periodic atomic structures with diffraction patterns consisting of spots.

Meyer's "cut-and-project" construction provides many examples of non-periodic distributions of this type, which have uniformly discrete support and dense point spectrum.

It has been conjectured, however, that if both the support and the spectrum are uniformly discrete sets, then the quasicrystal must have a periodic structure. We recently proved this conjecture in a joint work with Alexander Olevskii. Our approach is based on interaction of some problems in harmonic analysis and discrete geometry.

I will devote the first lecture to an exposition of the relevant background, and to a presentation of the above and related results. In the second and third lectures, the main aspects of the proofs will be discussed.

Additional talks:

Felix Pogorzelski, **The dynamics of Delone sets - an introduction**.

Delone sets in metric spaces are sets which are uniformly discrete and relatively dense. The talk is devoted to an introduction in the realm of locally compact, second countable groups. We shed some light on the naturally arising topological dynamical systems and their interplay with combinatorial properties of the point set in question. We conclude the talk with an elementary convergence result for dynamical systems which has applications to the spectral theory of discrete Schrödinger operators. The last part is based on joint work with Siegfried Beckus.

TBA, **Cut and project sets, with a drop of Diophantine approximation**.

Cut and project sets are, in many senses of the word, regular, but aperiodic point patterns obtained by projecting an irrational slice of the integer lattice to a subspace. In this talk I will describe the cut and project method and some of its key properties, focusing mainly on the Euclidean case. Time permitting, I will explain some recent developments of the theory: how to quantify the relationship between Diophantine approximation and repetitivity properties of cut and project sets, and how to use this connection to gain detailed information on speed of convergence to asymptotics, of frequences of patterns in cut and project sets. The talk is based on joint works with Alan Haynes, Antoine Julien and Jamie Walton.

Ilya Gringlaz, **Projections of cut and project sets**.

Lattices have the following "nice projections" property: Projections of a lattice along some directions are themselves lattices. Cut and project sets are in many ways similar to a lattice, but for most such sets, the projection along any direction is not discrete. We will define a similar "nice projections" property for cut and project sets, show a way to check whether a cut and project set has that property, show some interesting examples, and pose some open problems.

Yotam Smilansky, **Kakutani's splitting procedure for substitution partitions**.

In 1975, S. Kakutani introduced a splitting procedure which generates a sequence of partitions of the unit interval $[0, 1]$, and showed that this sequence is uniformly distributed in $[0, 1]$. We present generalizations of this procedure in higher dimensions, which correspond to constructions used when defining substitution and multiscale substitution tilings of Euclidean space. We prove uniform distribution of these sequences of partitions using new path counting results on graphs and establish Kakutani's result as a special case.

Faustin Adiceam, **Poor visibility in forests**.

We will survey some recent results dealing with the topic of visibility in so-called "forests". A typical question related to this topic is the following: "Suppose one stands in a forest with tree trunks of the same radius and no two trees centered closer than unit distance apart. Can the trees be arranged so that one can never see further than some distance V, no matter where one stands and what direction one looks in? What is the size of V in terms of?".

Boris Solomyak, **On the spectral theory of substitution systems**.

Tilings, in particular, those which arise from a substitution, have been used as models of quasicrystals. In particular, discrete diffraction spectrum is closely related to the discrete dynamical spectrum of the measure-preserving $\mathbb{R}^d$-action on the tiling space. The first part of the talk will be a review of this theory. In the second part of the talk, I will discuss some recent results, joint with A. Bufetov and A. Berlinkov, which concern the spectral properties of substitution systems that are not pure discrete, and thus it is of interest to determine the nature of the continuous spectrum.

Amos Nevo, **On the lattice point counting problem for irreducible lattices and some of its consequences**.

We will start by describing the general solution to the lattice point counting problem for irreducible lattices in algebraic groups, and then proceed to describe some of its consequences. These will include a counting asymptotics for cut-and-project sets, a solution to a generalization of Kazhdan's problem on equidistribution of dense isometry groups, and an effective ratio ergodic theorem for their actions. Based on joint work with Alex Gorodnik.

Organizers

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